IMPEDANCE MATCHING 101, V. 2.0

(MATCH ANYTHING TO ANYTHING)

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IMPEDANCE MATCHING BASICS

I. INTRODUCTION

Impedance matching is the basis for all RF design: low noise amplifiers, power amplifiers, and antenna design/combining. While there are numerous methods to match impedances, Smith Charts, computer simulations, etc., this paper addresses the basis for all of these other methods. It also demonstrates how impedance matching can be accomplished using simplified and straight forward techniques.

Multiple examples are given for typical RF design problems, and more complex circuits are simplified to demonstrate how these are simply extensions of the basic L-Matching circuit. Techniques are shown to achieve a broadband match to yield circuit performance over a wide band, and several examples are shown for unique situations such as the unequal power split.

II. THE BASIC ELEMENTS OF CIRCUITS

When discussing impedances, there are only three elements: the resistor, the inductor, and the capacitor. In all situations, the objective is to take an existing impedance and, using impedance matching, transform this impedance to a desired impedance. In some cases, such as matching a given impedance to 50 ohms, this is done in order that efficient power transfer can be achieved. If impedances are not matched, then excessive SWR's will occur, decreasing power or signal transfer, and often resulting in junction breakdown of the amplifying device.

Fifty ohm impedance is most often chosen as the characteristic impedance for Amateur Radio applications because standard transmission lines are manufactured to this impedance and test instrumentation is made to test at this impedance. Other impedances could just have easily been chosen as the standard, but 50 ohms is a convenient impedance for RF communications. In the television industry, the standard impedance used is 72 ohms, and for waveguide systems the standard impedance is 377 ohms, the impedance of waveguide. In all of these cases, the standard impedance is often referred to as Z_0 which can be any of these impedances depending on the sector of the industry being worked. For example, the center impedance of a Smith Chart is given as Z_0, but this is simply derived by dividing actual circuit impedances by the characteristic impedance. So, the center of the Smith Chart for communications systems is 1, but to obtain the actual impedance, this must be multiplied by 50. For that matter, any impedance on the Smith Chart, must be multiplied by 50 to obtain the actual impedance.

Before proceeding, it should be pointed out the inductive impedances are +J, meaning the current lags the voltage across it, and capacitive impedances are –J, meaning the voltage lags the current through it. Resistive impedances, have no vector direction as the current and voltage associated with a resistor are in phase. The "J" operator in the Electrical Engineering world is used to signify the imaginary operator "i", but since "i" is used for current in electronics, something else was necessary such that these two terms would not be confused.

Normally we think of inductance expressed in some factor of Henry's and capacitors in some factor of Farad's, but here we will use the impedance values of these components. Also, from math we know that $i = \sqrt{-1}$, so since we are using J in place of i, then $J = \sqrt{-1}$ and $J^2 = -1$. On the vector plot, +J is inductive, and –J is capacitive depicted in Fig. 1 below.

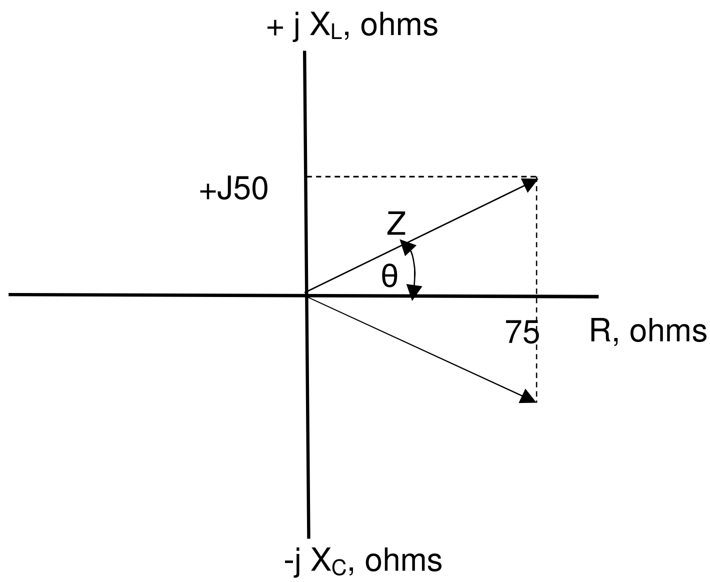
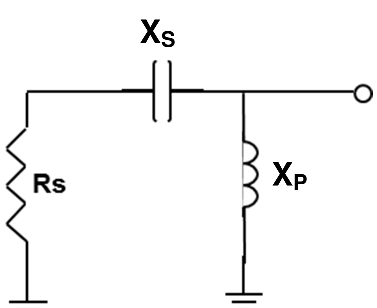


Figure 1 – VECTOR PLOT OF IMPEDANCES

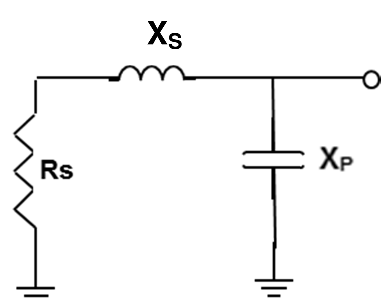
Just to make certain what is meant by impedance; there are three elements: the resistor, the inductor, and the capacitor. Only the resistor dissipates power, but the other elements can impact how much current flows in a circuit to pass through the resistive load. +J means the component is inductive or inductive reactance, and –J means the component is capacitive or capacitive reactance. Resistance is the "Real" component, and inductors and capacitors are the "Imaginary" components signified by +J or -J. The impedance of inductive reactance is dependent on the frequency of operation and is determined from +J = 2πfL, where 2π = 2(3.14) = 6.28, f is the frequency in Hz, and L is the inductance in Henrys. Similarly, the capacitive reactance is –J = 1/(2πfC), where 2π is defined above, f is still the frequency, and C is the capacitance value.

There are a few "shortcuts" we can in for RF design to ease the calculations of reactance values. First, inductive values are normally calculated in microhenries, (μH). When an inductance value is determined to 10^{-6} Henry's, we can write this as microhenries, and 2π is written as 6.28. Capacitive values are written in terms of picofarads or 10^{-12} Farads. When a calculation is determined to be $10^{-12,}$ this is written as microfarads. To further simplify calculations, instead of using 2π for capacitive calculations, use 1/(2π) which is 159 x 10^{-3}. This will become apparent later when calculations are actually performed.

Also, all matching circuits are an expansion of the "L" matching circuit. The L matching circuit looks like the letter L laid on its side ⌐ as shown. By that, we mean it either has a series capacitor and a parallel inductor to ground or a series inductor and a capacitor to ground. The two configurations are:



and



If we have two “L” matching circuits back to back, we develop the “PI” filter and the “T” filter. These are basically used for harmonic attenuation and not necessarily impedance matching although impedance matching can be incorporated into either circuit. The T and the PI circuits are shown below where the two inductors are paralleled for using one inductor.

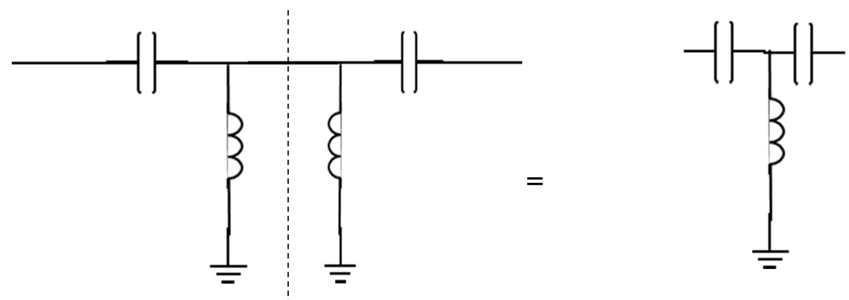


Figure 2 – T CIRCUIT FROM TWO “L’s”

Similarly, two “L” matching circuits can be used to configure a “PI” network BY replacing the two series inductors with one inductor as shown in Fig. 3 below.

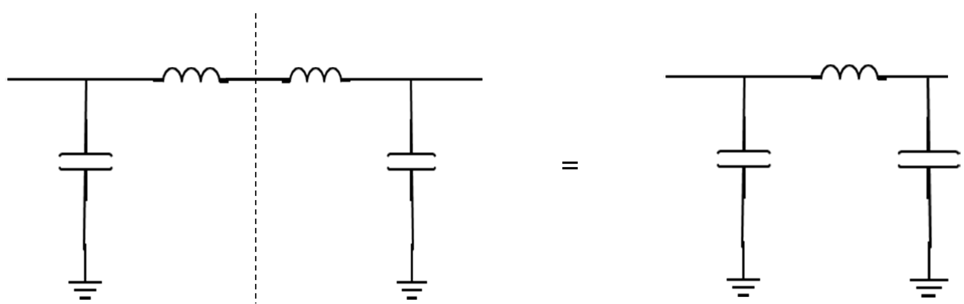


Figure 3 – PI CIRCUIT FROM TWO “L’s”

Also, to simply calculations, we always match from a low to a high impedance. If we have two “ L” sections back to back, either the series inductance or the parallel capacitance can be added to create the more familiar “PI” section or “T” section shown in Figures 2 and 3 above. These sections match to a high Q impedance to form a harmonic attenuation circuit.

III. THE CONCEPT OF "Q" AND IT'S FACTOR IN IMPEDANCE MATCHING

Q is defined as the stored energy over the dissipated energy or the resonant frequency divided by the bandwidth. For simple series and parallel circuits, the loaded Q is:

$Q = Xs/Rs = Rp/Xp$,

where Xs is the series reactance,

Rs is the series resistance,

Rp is the parallel resistance, and

Xp is the parallel reactance.

This is important because every series impedance has an equivalent parallel impedance, and the Q of each equivalency is the same. This is shown in Fig. 4 below.

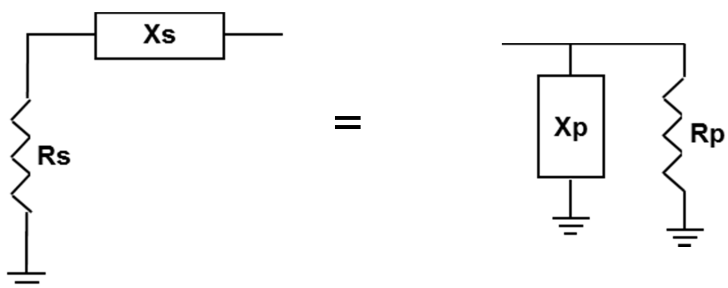


Figure 4 - SERIES/PARALLEL EQUIVALENT CIRCUITS

By knowing the Q of these two circuits, the series and parallel impedances are related by:

$$Rp = Rs\,(Q^2 + 1) \qquad \text{Equa. 1}$$

As an example, suppose we want to match 50 ohms to 200 ohms. If Rs is the 50 ohms and Rp is the 200 ohms, from Equa. 1 we determine the Q to be:

$$Q^2 + 1 = Rp/Rs = 200/50 = 4,$$

$$Q^2 = 4 - 1 = 3, \text{ and,}$$

$$Q = \sqrt{3} = 1.732$$

From Q = Xs/Rs, Xs = Q*Rs = 1.732*50 = 86.6. We can make Xs either inductive or capacitive, but which ever reactive element is used will cause the parallel equivalent reactance to be the same. Carrying one further step, since Q = 1.732 and Rp = 200 ohms, we can immediately determine that Xp = Rp/Q = 200/1.732 = 115.5 ohms. Since we made Xs inductive, Xp will also be inductive or +J 115.5 ohms. Now it is a simple matter to place the opposite value of reactance, -j 115.5 in parallel with the parallel circuit, and the +J 115.5 and –J 115.5 forms a parallel resonant circuit with infinite impedance. The infinite impedance in parallel with 200 ohms is still 200 ohms with no reactance. Hence, we have matched 50 ohms to 200 ohms with two components as shown in Fig. 5 below.

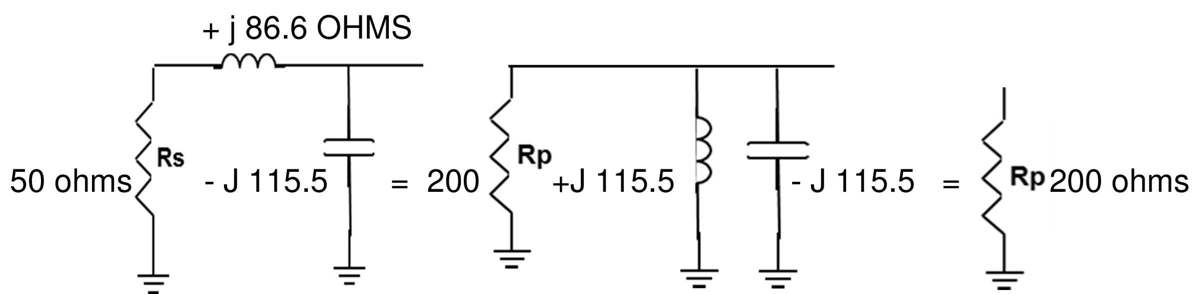


Figure 5 - L MATCHING TO MATCH 50 OHMS TO 200 OHMS

For achieving this match at 144 MHz, or 144 x 10^6 Hz, the component values are derived from $|X_L|$= 2π*f*L or L = $|X_L|$/2π*f = 86.6/2π*(144 x $10^{6)}$ = 95.8 x 10^{-9} = 95.8 nH.

Also, $|X_c|$= 1/(2π*f*C) or C = 1/(2π*f* X_c) = (159 x 10^{-3})/(144 x 10^6 *115.5) = 9.6 x 10-^{12} = 9.6 pf. Suppose the goal is to combine four antenna that have 50 ohm input impedances. For this case, four of the matching circuits could be connected together as shown below in Fig. 6.

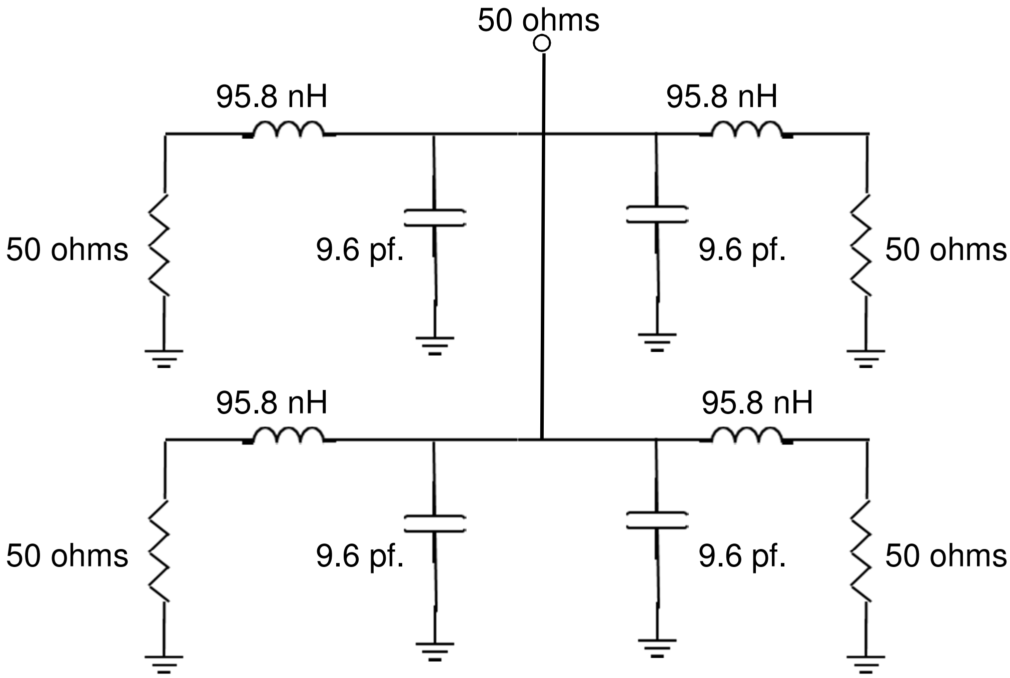


Figure 6 - FOUR WAY POWER COMBINER

IV. GENERAL GUIDELINES

If the impedance transformation is greater than 4:1 (such as matching 50 ohms to >200 ohms), then the circuit Q's become too high and component losses become excessive. Therefore, design a two-step match and use "complementary L matching sections. Also, the intermediate matching impedance should be the "geometric mean" impedance. Complementary L matching means that if a series L and parallel C is used for the first matching stage, the second stage should be a series C and a parallel L. The geometric mean impedance is determined by taking the square root of the starting and final impedances. This impedance presents to same percentage bandwidth going from the intermediate impedance to both the beginning and ending impedances.

For example, suppose we want to match 4 ohms to 50 ohms. Here, the impedance transformation is greater than 4:1 since 50/4 = 12.5. Therefore, use a two stage matching circuit, i.e. do the match in two steps. To determine the intermediate impedance, find the geometric mean impedance which is $(50 * 4)^{1/2} = 14.14$. Now we are matching 4 ohms to 14.14 ohms, a 3.5:1 impedance ratio and 14.14 ohms to 50 ohms, also a 3.5 ohm impedance ratio or 28.28% from the intermediate impedance, 14.14 ohms, to the lower impedance and 4 ohms and 28.28% to the higher impedance, 50 ohms, from 14.14 ohms. An example of this matching circuit is shown in the following section.

Also, it saves having to rearrange and remember addition equations if the match is always made from the lower impedance to the higher impedance. Then Equation 1 is all that is needed to perform the match. If it is necessary to match from a high impedance down to a lower impedance, make the design calculations from the low impedance to the high impedance and then connect the high impedance load to the matching circuit and look into the low impedance side of the match to see the lower impedance. Reciprocity holds in this case.

V. MATCHING EXAMPLES

A. Four Yagi Array Driver or PA Combiner

Figure 6 above shows how four 50 ohms loads can be combined to present a 50 ohms to a driving source or a preamp input for minimum noise figure. An example for using this circuit would be using a transmitter to drive four Yagi antennas while presenting a low SWR to the transmitter. In addition, the noise figure of the receiver preamp is designed for minimum noise figure when driven from a 50 ohm source. Anything other than 50 ohms will result in reflected power for the transmitter and a higher noise figure for the receiver and, hence, a lower Signal to Noise ratio for the received signal.

To feed 4 antennas at 2 meters, the four capacitors which are in parallel are replaced by one capacitor that is 4 times the value. Fig. 7 below shows this 4-way divider for the antennas. These antennas could be Yagi’s, Loops, or verticals. One thing to note is that the feedlines for all four antennas should be the same length of the same type of coax to maintain equal phasing.

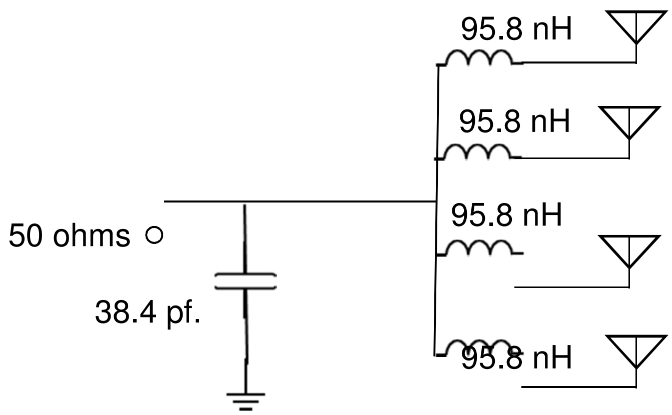


Figure 7- FEEDING 4 ANTENNAS

The circuit in Figure 6 is rearranged in Figure 8 on the following page to present a bit clearer picture for the 4-way divider used to combine four power amplifiers at 144 MHz. In this example, the 9.6 pf. capacitors which are in parallel are combined using one capacitor of four times the value or 38.4 pf, simplifying the design.

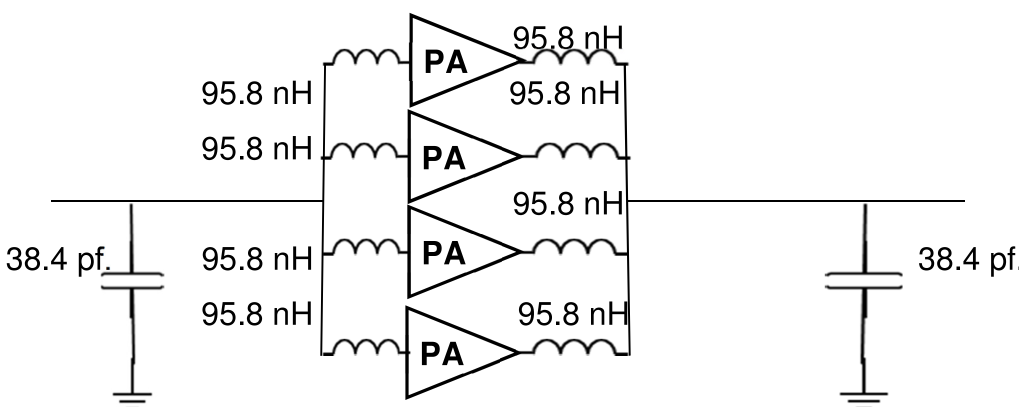


Figure 8 - FOUR WAY DIVIDER/COMBINER FOR DRIVING 4 PA'S

B. Complementary L Matching for Higher Impedance transformations

As mentioned above, suppose we want to match 4 ohms to 50 ohms, a typical input impedance for a VHF solid state power amplifier device. This match will be performed in two steps going from 4 ohms to 14.14 ohms, the geometric mean impedance, and a second step going from 14.14 ohms to 50 ohms. Also, complimentary L matching sections are used to provide increase bandwidth for the matching circuit. By that, if the first match uses a series L and a parallel C, the second match uses a series C and a parallel L. In this way, as one matching stage tends to go inductive as the frequency is changed, the second matching stage goes capacitive, thereby maintaining the match over a wide frequency range. Below is an example of this for the 2 meter band.

First, a matching circuit is designed for 4 ohms to 14.14 ohms. Since $Rp = Rs(Q^2 + 1)$, Equa. 1, $Q = \sqrt{(14.14/4) - 1} = 1.59$. Since Q = Rp/Xp, therefore Xp = 14.14/Q = 14.14/1.59 = 8.9. Q also = Xs/Rs, so Xs = Rs * Q = 4 * 1.59 = 6.36 ohms. If a series inductor is chosen (it could be a series capacitor of the same impedance value), the parallel equivalent circuit is also inductive with a value of +J 8.9 ohms.

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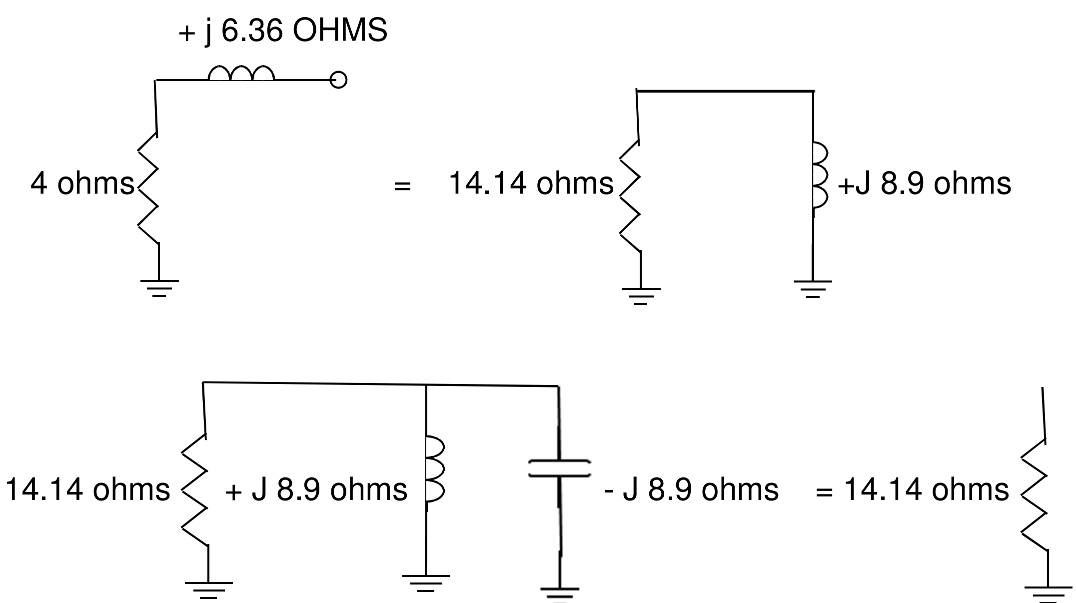


Figure 9 - 1st STAGE MATCH

The 4 ohms in series with + J 6.36 ohms yields a parallel equivalent circuit of 14.14 ohms in parallel with + J 8.9 ohms, so placing a – J 8.9 ohms, a capacitor, in parallel with this circuit results in 14.14 ohms resistive in parallel with + J 8.9 ohms and – J 8.9 ohms. The + J 8.9 ohms and – J 8.9 ohms in parallel results in an infinite impedance which is, in turn, in parallel with 14.14 ohms resistive which is 14.14 ohms resistive.

The next stage is to match this 14.14 ohm resistive value to 50 ohms. For the first stage matching, a series inductor was added along with a parallel capacitor to resonate the inductance. For the second stage match, a series capacitor will be added along with a parallel inductor to get to the desired 50 ohm resistive value. This matching circuit is shown below with a + J 31.4 ohm parallel inductor to resonate the – J 31.4 ohms capacitor, leaving the 50 ohms resistive.

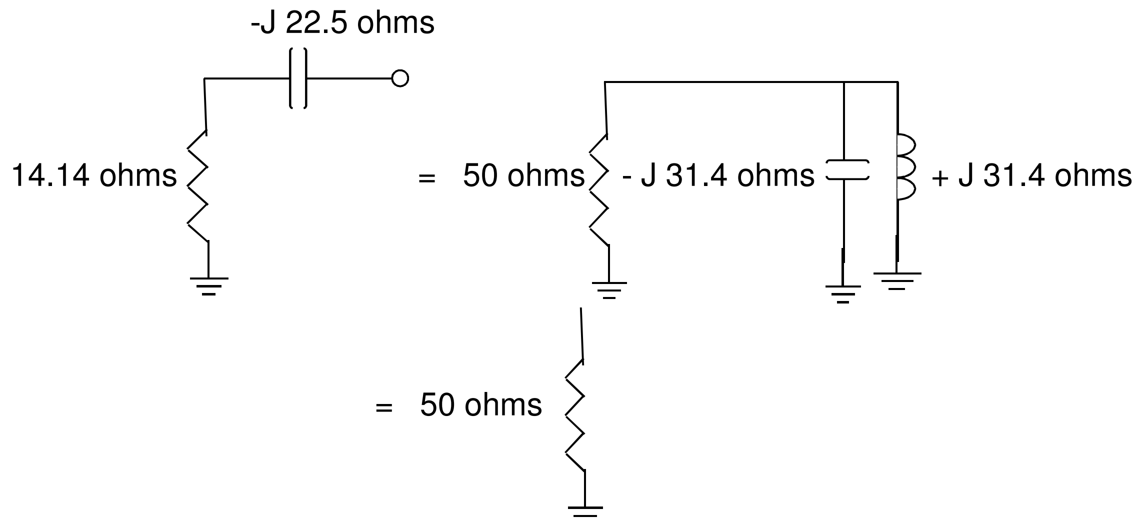


Figure 10 - MATICHING COMPONENT FOR 14.14 TO 50 OHMS

The total matching circuit, both stages, is shown below in Fig. 11. Again, the first stage uses a series inductor and parallel capacitor while the second stage uses a series capacitor and a parallel capacitor in order to increase the bandwidth of the match. As one stage goes inductive with a change in frequency, the other stage goes capacitive tending to compensate each other.

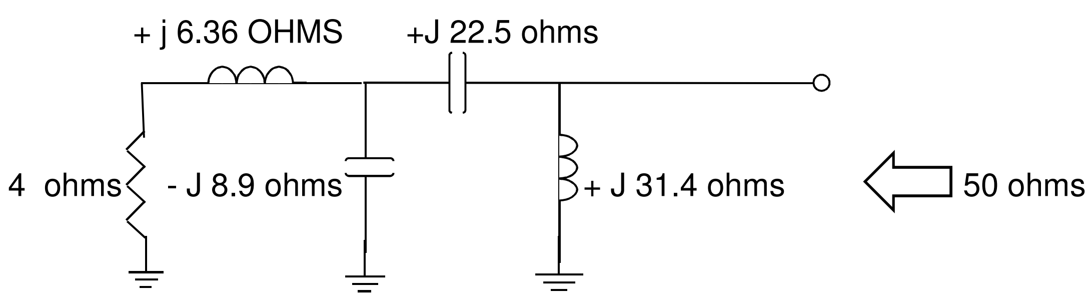


Figure 11 - TWO STAGE MATCHING FOR 50 OHMS

Calculating the component values for 144 MHz yields the circuit shown in Figure 12.

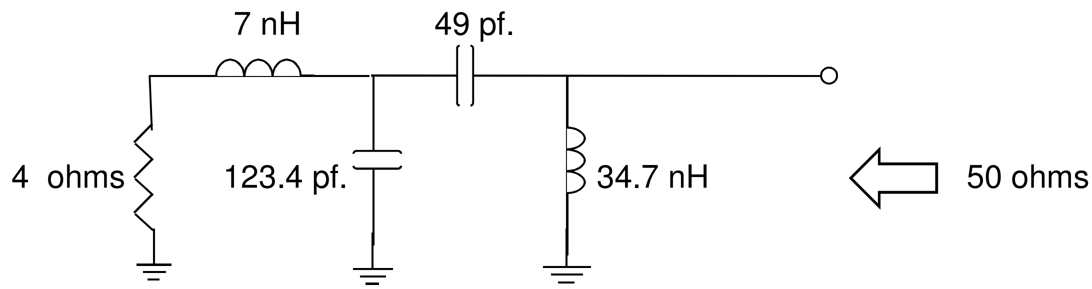


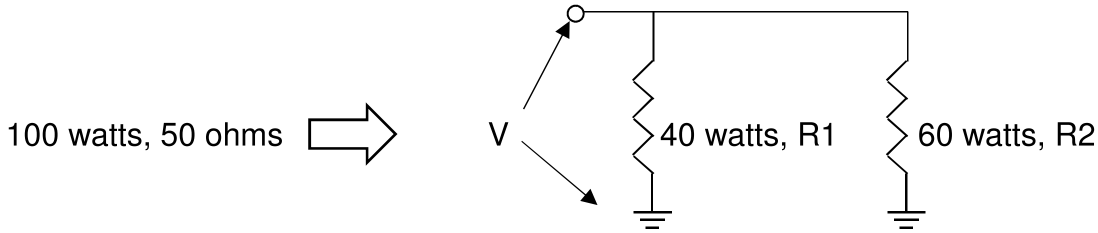
Figure 12 - COMPONENT VALUES FOR TWO STAGE MATCHING CIRCUIT

C. Unbalanced Power Divider

Another problem encountered on occasion might be the unbalanced power divider. This situation occurs when driving a high power amplifier that required less drive than the output of the transceiver. Occasionally the operator forgets to turn down the transceiver output to the level necessary to drive the amplifier, or sometimes a power output adjustment isn't even available. If the output amplifier is overdriven, it results in a highly distorted signal on the band or, even worse, a blown output amplifier.

An approach to solve this issue is to dump a portion of the driving power into a 50 ohm, high power termination with the remaining drive power going to the input of the amplifier. As an example, suppose we have a 6 meter KW. amplifier that requires 40 watts of drive and our transceiver puts out 100 watts. The problem is to transfer 40 watts of power to the amplifier and the remaining 60 watts to a 50 ohm termination and still provide a low SWR match for the transceiver.

First, it is necessary to determine the impedances necessary for the two power ratios. The problem is simplified to look like two resistors in parallel with one resistor getting 40 watts of power and the other resistor getting 60 watts of power and the parallel combination of the two resistors to be 50 ohms.



From $P = V^2/R$, the power for R1 is 40 watts = V^2/R1, and the power for R2 is 60 watts = V^2/R2. Since the voltage across both resistors is the same, rewriting the above yields V^2 = 40*R1 = 60*R2 or <u>R1 = (60/40)*R2 = 1.5 R2</u>. Also, from two resistors in parallel, the parallel combination of R1 and R2 in parallel is 50 ohms and is:

$$R = 50 \text{ ohms} = \frac{R1 * R2}{R1 + R2} \qquad \text{Equa. 2.}$$

Since R1 = 1.5 R2, substituting 1.5 R2 for R1 yields:

$$R = 50 \text{ ohms} = \frac{1.5R2 * R2}{1.5R2 + R2}, \text{ and}$$

$$1.5\ R2^2 = 125\ R2, \text{ or}$$

$$1.5\ R2 = 125, \text{ and}$$

$$R2 = 83.3 \text{ ohms.}$$

Since R1 = 1.5 R2, R1 = 125 ohms.

Checking the parallel combination results in:

$$\frac{125 * 83.3}{125 + 83.3} = \frac{10{,}412.5}{208.3} = 50 \text{ ohms} \qquad \text{Equa. 3.}$$

So, now the problems is to match R1, the amplifier input, from 50 ohms to 125 ohms and the 50 ohm termination to 83.3 ohms and connect these two impedances in parallel.

Referring back to Equa. 1, for R1 = 125 ohms = 50 (Q^2 + 1), Q = 1.22 and, using a capacitor for the impedance transform, the capacitive reactance is –J 61.2 ohms.

At 50 MHz, the capacitance for –J61.2 ohms is 51.9 pf. or very close to 52 pf. The resulting parallel capacitive impedance is –J 102.1 ohms which normally is tuned to resonance with a +J 102.1 ohm inductor. However, proceeding with the calculation to transform 50 ohms to 83.3 ohms to step up the impedance of the 50 ohm termination, following the same process yields a series reactance of +J 40.8 ohms. This produces a parallel equivalent of 83 ohms in parallel with +J 102.1 ohms.

Normally, a –J 102.1 ohms would be used to resonate the +J 102.1 ohms, but an observation of the matching circuit to match 50 ohms to 125 ohms shows that +J 102.1 ohms is the inductance necessary to resonate the –J 102.2 ohms obtained when matching 50 ohms to 125 ohms. So, all of these components are connected in parallel using the + reactance for one match to tune out the – reactance for the other match. The total circuit is shown below in Fig. 13 along with the resulting components when these two components

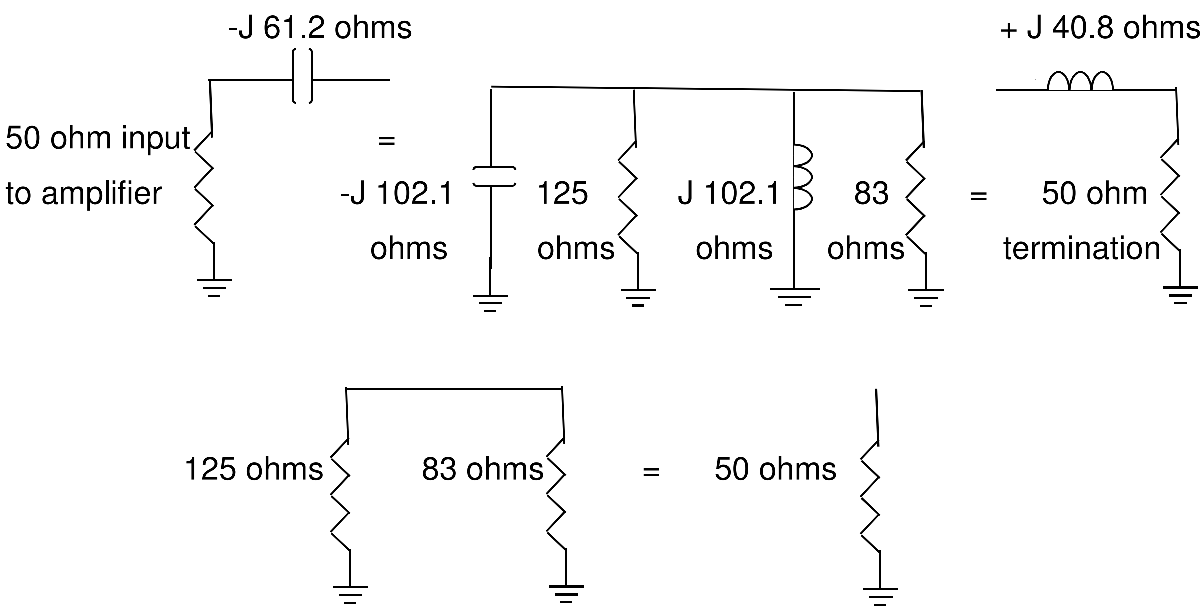


Figure 13 - MATCHING CIRCUIT FOR UNBALANCED POWER SPLIT

So, the final matching circuit is just the series capacitor and the series inductor connected to their respective loads. This produces the unbalanced power split and maintains a 50 ohm load for the driving transceiver.

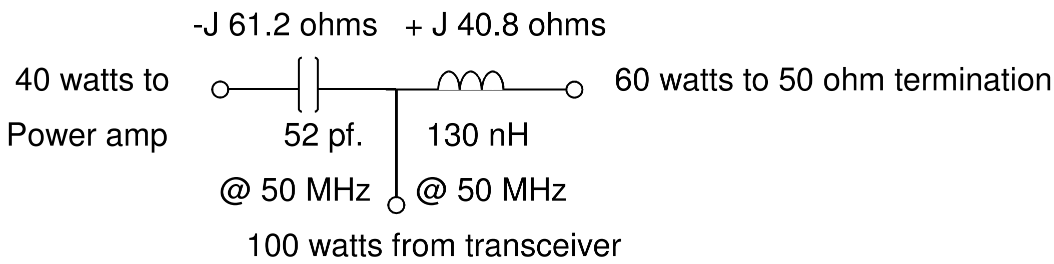


Figure 14 - FINAL UNBALANCED DIVIDER CIRCUIT

D. 90 Degree Signal Phase shifter For I and Q Signal Generation

Another design requirement for Amateur operation might be the generation of in-phase, I, and quadrature phase (90 deg.), Q, signal generation. This would be used for signal processing and also for developing two signals 90 degrees out of phase to generate circular polarization.

Many MMIC's are available from sources such as Mini Circuits that are broadband amplifiers with a 50 ohm input impedance. A simple approach to generate two outputs that are 90 degrees out of phase is to use two devices and match both to 100 ohms, one with an inductor, and the other with a capacitor. If both 50 ohm inputs are matched to 100 ohms using a +J 50 ohm inductor and a –J 50 ohm capacitor, the two inputs connected in parallel results in the circuit shown in Fig. 15 below.

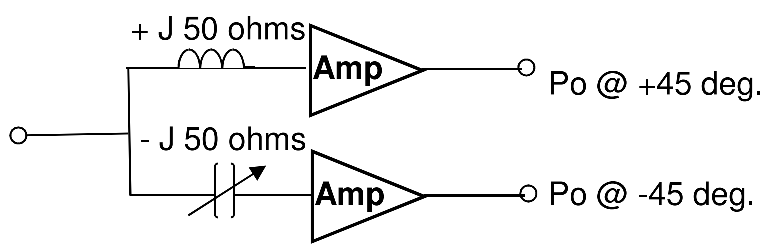
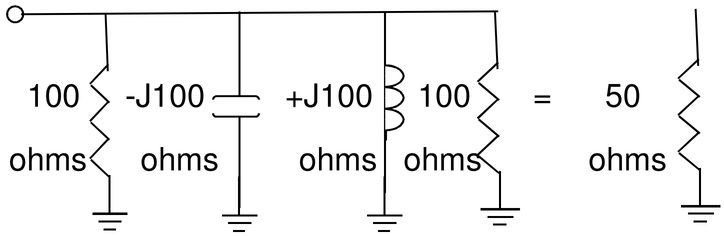


Figure 15 - INPUT TO OBTAIN QUADRATURE OUTPUTS

The input of this configuration appears as follows.



Making the series input capacitor variable provides the capability to adjust the two outputs to exactly 90degrees out of phase. The same approach can be used to generator two output from power amplifiers that are 90 degrees out of phase for applications such as the generation of circular polarization. This is shown below.

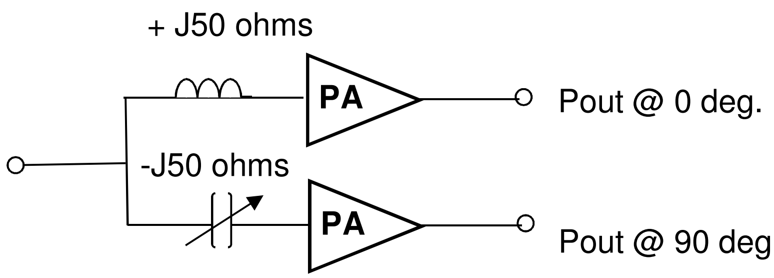


Figure 16 - CIRCUIT TO GENERATE TWO 90 DEG. OUTPUTS FOR CIRCULAR POLARIZATION

E. 1296 MHz QUADRATURE COUPLER

Multiple applications, both receiving and transmitting, require a quadrature coupler to produce I-phase, I, and Quadrature-phase, Q, signal generation. Examples are: Software Defined Radios, Single Sideband generation, Circular Polarized antennas, Doherty amplifiers, etc. For the case addressed here, a method for generating I and Q signals for circular polarization will be presented.

SM6FHZ and SM6PGP have presented a unique method for circular pattern generation for feeding a parabolic dish antenna. What really makes this approach attractive is that the quadrature couplers for receiving come AFTER the low noise preamplifier (LNA), thereby avoiding the introduction of loss before the LNA. Similarly, the transmit quadrature coupler is inserted before the power amplifiers avoiding loss there also. See Fig. 17 for this block diagram.

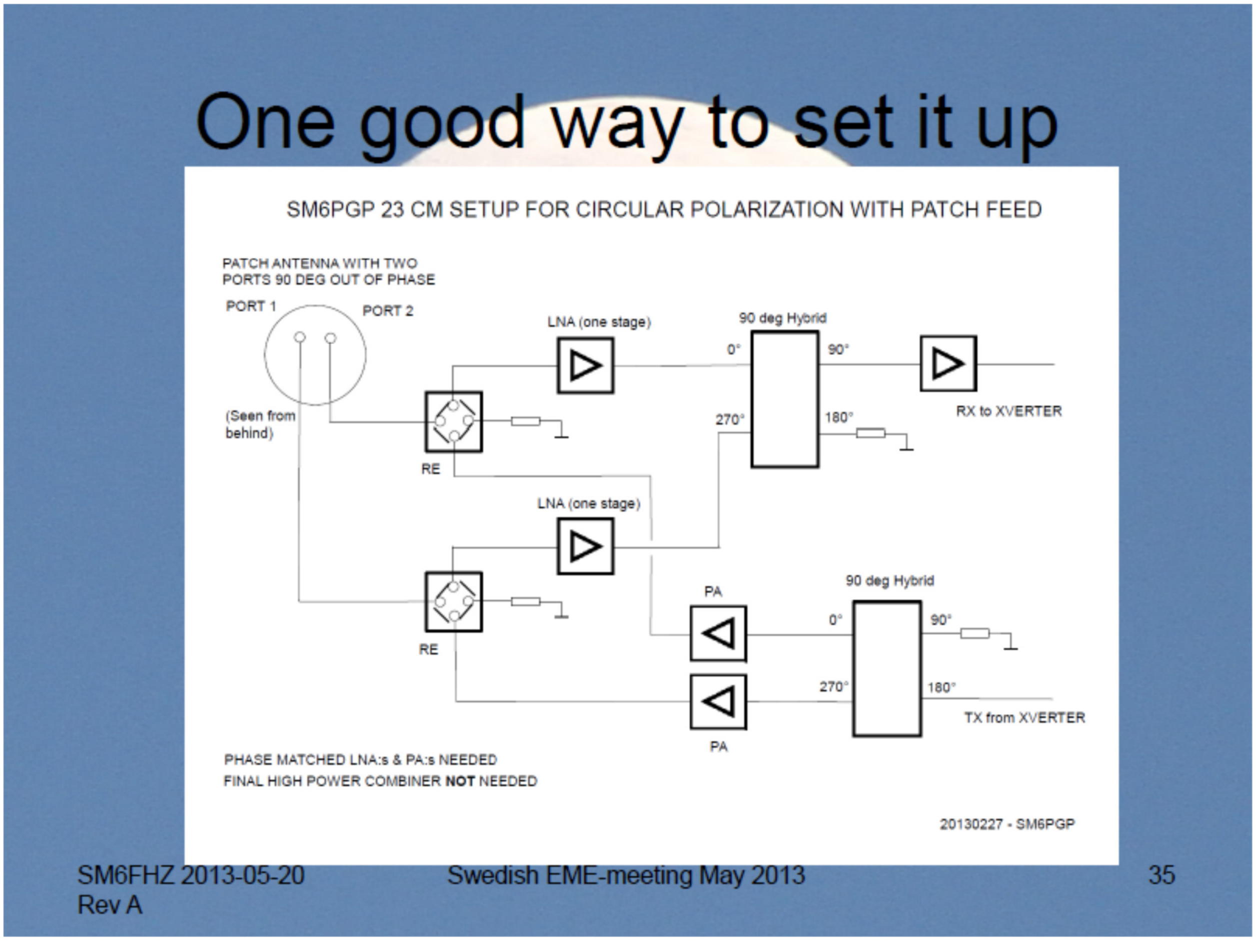


Figure 17- CIRCULAR POLARIZATION GENERATION

While this approach requires two LNA's, two transfer relays, two power amplifiers, and two 90 deg. hybrid couplers, and phase balancing the transmission lines between the LNA's and PA's, it avoids introducing losses in these critical paths. It also negates the requirement for a high power combiner as is often used to generate the desired power levels for EME. Typically, the LNA's, PA's, and transfer relays shown in Fig. 17 are purchased items, although some hams do build these items themselves. The 90 deg. hybrids are also often purchased, and these hybrids incorporate a "difference" resistor to compensate for any imbalance. As is the case for any resistor, there is an associated power limit for the difference resistor.

One way to avoid difference resistor limitation is to design a 90 deg. coupler that does not have a difference resistor. An example of this is shown on the following page in Fig. 18.

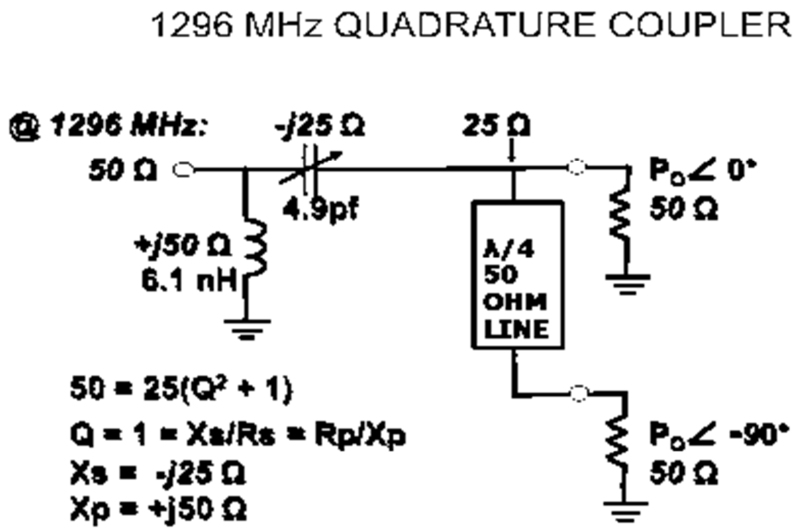
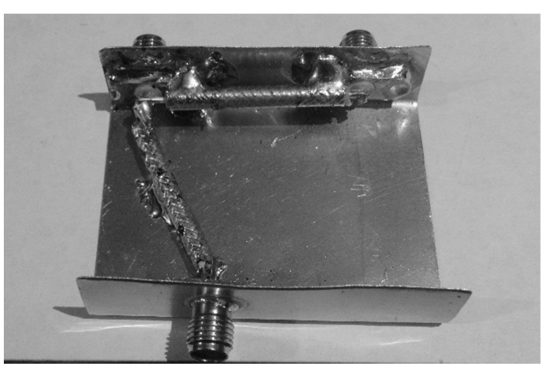


Figure 18 - 1296 MHz QUADRATURE HYBRID

In this arrangement, two 50 ohm loads are connected in parallel, but one of the signal paths is via a λ/4, 50 ohm transmission line. When the two 50 ohm impedances are combined, they produce an impedance of 25 ohms. Using the procedure previously described to match this 25 ohms to 50 ohms, an L-matching circuit consisting of a 4.9 pf. series capacitor and a 6.1 nH inductor transforms the 25 ohms to 50 ohms. Fig. 19 below depicts this circuit.



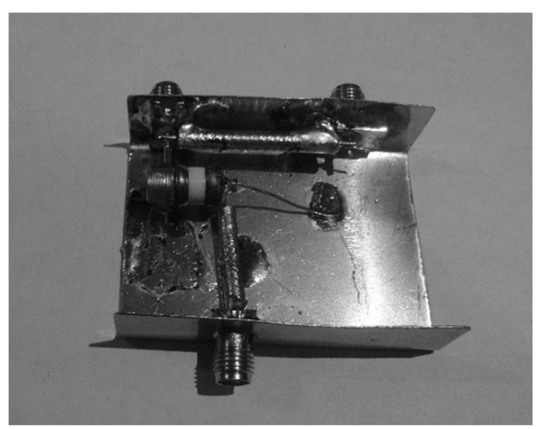


Figure 19 - CIRCUIT IMPLEMENTATION OF 90 DEG. COUPLER

The first step is to adjust the length of the quarter wave line to be exactly 90 degrees including the effects of the SMA connector at the end of the line. This line is soldered in place, and a temporary through line is placed between the 0 deg. output connector and the input connector. With a spectrum analyzer and signal generator connected to the ends of the through line, the frequency is varied until the length of the 90 deg. line is adjusted until a null is achieved at 1296 MHz. This is exactly ¼ λ or 90 deg. The through line is then replaced with the matching circuit to match 25 ohms to 50 ohms, and the quadrature coupler is completed.

VI. CONCLUSION

Impedance matching is the basis of all circuit design. This paper has demonstrated how to calculate the component values to match impedances and has provided several examples in which impedance matching will provide the desired circuits for accomplishing a goal.